CONTINUUM MODEL FOR SPACE DEBRIS EVOLUTION WITH ACCOUNT OF COLLISIONS AND ORBITAL BREAKUPS

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Abstract. The paper discusses the mathematical modeling of long-term orbital debris evolution taking into account mutual collisions of space debris particles of different sizes. Investigations and long-term forecasts of orbital debris environment evolution in low Earth orbits are essential for future space mission hazard evaluation and for adopting rational space policies and mitigation measures. The paper introduces a new approach to space debris evolution mathematical modeling based on continuum mechanics incorporating partial differential equations. This is an alternative to the traditional approaches of celestial mechanics incorporating ordinary differential equations to model fragments evolution. The continuum approach to orbital debris evolution modeling has essential advantages for describing the evolution of a large number of particles, because it replaces the traditional tracking of space objects by modeling the evolution of their density of distribution.

Keywords: atmospheric drag, collisions, current debris environment, debris evolution, fragmentation, production

Nomenclature

Latin symbols

b_j^α	fragment number *versus* mass distribution parameter in breakup of the αth fragment on forming smaller fragments belonging to a jth phase
C_f^j	the atmospheric drag coefficient for the upper rarefied layers
D	characteristic size of fragments
d_j	characteristic size of fragments belonging to the jth phase
e	specific elastic internal energy for material
F_d	mean cross-sectional area of collisions of an object of size D with particles from the range of (d_1, d_2)
F_{Dd}	mean cross-sectional area of collisions of objects in the size range (D_1, D_2) with particles from the range of (d_1, d_2)
$f(d) = dk(d)/dd$	the slope of fragments distribution coefficient growth
G	the universal gravity constant
$H(h, t)$	is the scale height of a uniform atmosphere at the altitude h
h	altitude

h_j	the perigee altitude of the orbit
i_j	inclination of the orbit
$J_i(z)$	the ith Bessel function
$K(D, d_1, d_2)$	the averaged number of collisions of a single spacecraft of diameter D with particles in the size range of (d_1, d_2)
k_b	ballistic coefficient
$k(d)$	particle size distribution coefficient
k_F	the shape coefficient
M	the mass of the Earth
M_j	reference mass of a fragment belonging to jth phase $(M_j \leq M_j)$
m_j^α	mass of fragments formed in destruction of two colliding fragments $(\alpha = 1, 2)$
$N_j(t, r)$	the number of debris particles of the jth phase per altitude spherical layer of thickness Δh
$\dot{N}_j$	the rate of change of the number of particles per altitude layer due to external sources and fragmentation in collisions
N_p	the number of selected phases
$n(h, h + \Delta h)_{cat}$	is the number of the catalogued objects within the altitude range $(h, h + \Delta h)$
n_{crit}	critical number density
$n_{j\,op}, n_{j\,ex}$,	the rates of particles number per unit volume growth and/or decrease due to external sources: operational debris, fragmentation in explosions
$p(\ldots)$	probability distribution function
$p(d)$	the density of distribution of particles mean diameter d
$p(t, A)$	the azimuthal distribution of SO at time moment t
$\bar{Q}$	the average SO flux through a unit cross-section of a space vehicle per revolution
r	radial co-ordinate in a spherical system
S	the satellite cross-section area
S_j^α	the total surface area for fragment m_j^α originating in fragmentation of M_α
s_j^α	the area of new surface for fragment m_j^α arising in fragmentation of M_α
t	time
u	specific internal energy for material
$V_j = -W_j$	radial velocity of altitude loss
V_{rel}	the relative velocity of a particle with respect to the given satellite
$\vec{v}_j(\vec{x})$	local orbital velocity of fragments belonging to the jth phase
$W_j(t, r) = (dr/dt)_j$	the radial velocity of altitude loss for particles (sedimentation velocity)

Greek symbols

α	volume concentration of debris fragments
χ	generalized characteristic of a space debris fragment
Δh	spherical layer thickness
δ	thickness of a flat debris element
ε_j	eccentricity
Γ	coefficient characterizing the reference mass subjected to fragmentation in collisions
γ_α	specific energy required for producing a unit of free surface in destruction of the α-th fragment material
η_j	the rate of self-cleaning due to burn up of fragments sedimenting in the dense layers of the atmosphere;
Λ	material constant characterizing fragmentation process
$\theta_j^\alpha = s_j^\alpha / S_j^\alpha, 0 < \theta_j^\alpha < 1$	the share of breakup new free surface among overall fragment surface
θ	latitude
$\rho_a(r, t)$	atmospheric density
ρ_j	number density of particles of the jth phase per volume unit
ρ_j^0	actual density of a particle
Ω	geographical longitude
$\xi_j, 0 < \xi_j < 1$	factor characterizing the deviation of the actual value for a fragment shape coefficient from its maximal value for a sphere
ψ_{jk}	the number of particles transferred from the kth to the jth phase per time unit due to fragmentation in collisions
ζ	specific dissipation

1. Introduction

The present model is based on the continuum mechanics approach (Smirnov et al., 1993). To simulate the space debris environment containing fragments differing greatly in mass, velocity and orbital parameters, the multiphase continua approach was introduced distinguishing classes of fragments possessing similar properties. Under this approach the evolution equations contain a number of source terms responsible for the variations of different fractions of orbital debris population due to fragmentation and collisions. Those source terms were developed based on the solution of a high-velocity collision and breakup problem (Kiselev, 2001; Smirnov et al., 1997). The Russian Space Debris Prediction and Analysis (SDPA) model (Nazarenko, 1993, 1996, 1997), developed using the continua approach, served as the basis for the present study. The model used the averaged description for the sources of space debris production and took into account collisions of debris fragments of different sizes (including non-catalogued ones) that could lead not only to debris self-production but also to a self-cleaning of the low Earth orbits (LEO). Self-cleaning of

LEO takes place due to the influence of atmospheric drag, which reduces the speed of fragments causing their sedimentation and burning up in the dense layers of the atmosphere. Smaller fragments are much stronger influenced by the atmospheric drag than the big ones.

2. Mathematical Model

Essential differences in debris particles sizes and thus collision consequences means that it is necessary to introduce a number of 'phases' or 'mutually penetrating continua' into the model, each phase being characterized by its own density of distribution. The particles could be assembled into groups ('phases') based on the following attributes: their characteristic size d_j; the perigee altitude of the orbit h_j, eccentricity ε_j; inclination of the orbit i_j; ballistic coefficient k_b, which means, that particles having the relevant characteristics $\chi = (d_j; h_j; \varepsilon_j; i_j; \ldots;$ etc.) in the range $\chi \in [\chi_j - \Delta\chi; \chi_j + \Delta\chi)$ belong to the jth phase, where $\chi_{j+1} = \chi_j + 2\Delta\chi$. The number density of particles of the jth phase per volume unit ρ_j evolution can be determined by the following equation (Smirnov et al., 1993):

$$\frac{\partial \rho_j}{\partial t} + \mathrm{div}\rho_j \vec{v}_j = \sum_{k=1}^{N} \psi_{jk} + n_{j\,\mathrm{op}} + n_{j\,\mathrm{ex}} - \eta_j, \tag{1}$$

where ψ_{jk} is the number of particles transferred from the kth to the jth phase per time unit due to fragmentation in collisions ($j, k = 1, \ldots, N_p$); $n_{j\,\mathrm{op}}, n_{j\,\mathrm{ex}}, \eta_j$ is the the rates of particles number per unit volume growth and/or decrease due to external sources: operational debris, fragmentation in explosions, self-cleaning due to burning up of fragments sedimenting in the dense layers of the atmosphere; $\vec{v}_j(\vec{x})$ is the local velocity of the jth phase, which is determined as a mean mass velocity for the fragments of the jth phase located in a macroscopically small volume with the centre defined by the vector $\vec{x}$.

Averaging Eq. (1) in longitude Ω and latitude θ gives the following form of the model equation:

$$\frac{\partial N_j}{\partial t} = -W_j \frac{\partial N_j}{\partial r} - N_j \frac{\partial W_j}{\partial r} + \dot{N}_j, \tag{2}$$

where $N_j(t, r)$ is the number of debris particles of the jth phase per altitude spherical layer of thickness Δh; $W_j(t, r) = (\mathrm{d}r/\mathrm{d}t)_j$ is the radial velocity of altitude loss for particles (sedimentation velocity); $\dot{N}_j$ the rate of variation of particles number per altitude layer due to external sources, given by the following relationship:

$$\begin{aligned}\dot{N}_j(t, r) = \int_r^{r+\Delta h} \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} \Bigg(& \sum_{k=1}^{N_\mathrm{p}} \psi_{jk}(t, r, \Omega, \theta) - \eta_j(t, r, \Omega, \theta) \\ & + n_{j\,\mathrm{op}}(t, r, \Omega, \theta) + n_{j\,\mathrm{ex}}(t, r, \Omega, \theta) \Bigg) \cdot r^2 \cos\theta \, \mathrm{d}\Omega \, \mathrm{d}\theta \, \mathrm{d}r \\ = \Psi_j + N_{j\,\mathrm{op}} + N_{j\,\mathrm{ex}}. &\end{aligned}$$

Equation (2) could help us to introduce several important definitions. Let us name the first critical number density of space objects in LEO $n_*(r)$ the number density, which being surpassed brings to the overall growth of number of fragments in the altitude layer:

$$\frac{\partial N_j}{\partial t} = -W_j \frac{\partial N_j}{\partial r} - N_j \frac{\partial W_j}{\partial r} + \dot{N}_j > 0,$$

which means that particles production and self-production $\dot{N}_j = \Psi_j + N_{j\,\mathrm{op}} + N_{j\,\mathrm{ex}}$ surpasses particles removing from the layer in the process of self-cleaning due to sedimentation. Thus determined value of critical number density would be a function of human space activity and the adopted space policy.

One could define the absolute critical number density $n_{\mathrm{crit}}(r)$, on reaching which the debris self-production process surpasses self-cleaning independently of human space activities, that is, even if no additional fragments are being added

$$-W_j \frac{\partial N_j}{\partial r} - N_j \frac{\partial W_j}{\partial r} + \Psi_j > 0.$$

The process characterized by debris self-production surpassing self-cleaning got the definition of 'cascade effect' in literature.

The sedimentation velocity in Eq. (2) can be determined by formula (Smirnov et al., 1993):

$$\dot{r} = W_j = -\frac{3}{2} \frac{C_f^j \rho_{\mathrm{a}}(r,t)}{\rho_j^0 d_j} \sqrt{GMr}, \tag{3}$$

where G is the universal gravity constant and M is the mass of the Earth, ρ_j^0 is the actual density of material of a particle, C_f^j is the atmospheric drag coefficient for the upper rarefied layers, the function $\rho_{\mathrm{a}}(r,t)$ (atmospheric density) could be obtained from one of the models of a standard atmosphere, or from its approximations

$$\rho_{\mathrm{a}}(r,t) = \rho_{\mathrm{a}}(r_0,t) \exp\left(-\int_{r_0}^{r} \frac{\mathrm{d}r}{H(r,t)}\right), \tag{4}$$

where $H(r,t)$ is the scale height of a uniform atmosphere at the altitude r; $\rho_{\mathrm{a}}(r_0,t)$ is a known density at the altitude r_0. Substituting Eqs. (3) and (4) in Eq. (2) one obtains

$$\frac{\partial N_j}{\partial t} = -W_j \frac{\partial N_j}{\partial r} + \frac{N_j W_j}{H}\left(1 - \frac{H}{2r}\right) + \dot{N}_j. \tag{5}$$

Introducing positive sedimentation velocity $V_j(t,r) = -W_j(t,r) > 0$ and assuming $(H/2r) \ll 1$ brings the Eq. (5) to the form used in the SDPA model

$$\frac{\partial N_j(t,h)}{\partial t} = V_j\left(\frac{\partial N_j(t,h)}{\partial h} - \frac{N_j(t,h)}{H}\right) + \dot{N}_j, \tag{6}$$

where h is the perigee altitude. For elliptical orbits with small eccentricity sedimentation velocity could be determined by the formula adopted for the SDPA model:

$$V_j = \frac{3}{2} \frac{C_f^j \rho_{\mathrm{a}}(r,t)}{\rho_j^0 d_j} \sqrt{GMr}(1-\varepsilon_j)\exp(-z)F(z),$$

where r is the semimajor axis of the orbit, ε_j is the eccentricity; $z = \varepsilon_j r/H$; function $F(z)$ for $z \ll 1$ could be determined by the following formula

$$F(z) = [J_0(z) - J_1(z)] + \varepsilon_j \left[J_1(z) - \tfrac{1}{2}(J_0(z) + J_2(z))\right],$$

where $J_i(z)$ is the ith Bessel function.

Equation (6) could be used to describe the evolution of particles having elliptic orbits as well. If the eccentricity of the orbit is rather high, a particle can cross several altitude layers in its motion. Then $N_j(t, h)$ in (6) should be considered as a number of particles *versus* perigee altitude distribution, or the density of the perigee altitude distribution for objects from the jth group at time t.

The last term in Eq. (6) reflects the particles flux to the jth phase from the external sources and due to fragmentation in collisions. The external sources should be introduced into the model as additional governing parameters, and the role of those terms would be dependent on the assigned values of the governing parameters, predicting the future human space activities.

Now we will concentrate our attention on the role of the internal mechanisms of debris production in collisions. To understand the role of the term $\sum_{k=1}^{N_p} \psi_{jk}$ in Eqs. (1), (2) and (6) one could regard the marginal case $N_p = 1$. Under this assumption, there exists only one phase ($\psi_{11} \neq 0$). The particles produced in collisions should be of a smaller size and normally join a different phase. For a one-phase model the new-born particles are preserved within the phase, thus decreasing the mean diameter of fragments d. Then the rate of fragmentation could be introduced by the formula (Smirnov et al., 1993):

$$\psi_{11} = \pi d^2 \rho^2 v_\tau \Psi, \tag{7}$$

where Ψ is the total average number of debris particles generated by one collision to stay in orbit for sufficient time to be taken into account for long-term forecasts, v_τ is the tangential relative velocity.

The initial stages of orbital debris evolution are characterized by a rarefied debris environment and the collisions of particles are also rather rare and the average diameter of particles remains stable. The growth of the number of particles due to collisions is proportional to the squared number density (7): $\psi_{11} \sim \rho^2$. After the debris number density increases above the critical value the collisions happen to take place more frequently, the particles number density grows rapidly and their mean diameter decreases. Assuming the new space programs to be stopped by that time one could expect the volumetric content of debris in orbit to be stable (disregarding its gradual decrease due to sedimentation):

$$\begin{aligned}
\alpha &= \frac{\pi d^3}{6}\rho = \text{const} \quad \text{(for compact elements without empty space inside);} \\
\alpha &= \tfrac{1}{4}\pi d^2 \delta\rho = \text{const} \quad \text{(for plane elements, which characteristic size} \\
&\quad \text{(in one direction the thickness } \delta\text{) is much less than the} \\
&\quad \text{characteristic size in orthogonal directions } (d),\ \delta \ll d). \qquad (8)
\end{aligned}$$

From Eqs. (7) and (8) it follows that the rate of debris population growth is proportional to $\rho^{4/3}$ for compact elements and $\sim\rho$ for plane elements. The effect of atmospheric drag increases with the decrease of particles size thus increasing the rate of sedimentation of small particles, Eq. (3) thus contributing to the increase of LEO self-cleaning effect (α decrease).

Equation (6) makes it possible to obtain an estimate of the critical number density leading to the cascade effect of debris self-production within a definite altitude layer.

Assuming the number of particles per unit layer to be uniformly distributed, we have two competing mechanisms governing the variation of the number of particles within the layer: particles production and self-production on one hand, and self-cleaning due to sedimentation on the other. Then the criterion for the growth of the number of particles will be the following:

$$\frac{NW}{H(r)} + \dot{N}_\Sigma > 0. \tag{9}$$

The rate of growth of particles number $\dot{N}_\Sigma = \psi_{11} + \dot{N}$ is the sum of self-production in collisions and production due to external sources. As the production due to external sources is determined mostly by the space policy, this value could be regarded as a slowly varying function with a minor contribution on the eve of the cascade process in comparison with the self-production term. Thus neglecting the external production terms and substituting the expressions (3) and (7) for sedimentation and fragmentation velocities one could transform the inequality (9) as follows:

$$4\pi^2 r^2 d^2 \rho^2 v_\tau \Psi \Delta h > \frac{3}{2}\frac{N}{H(r)}\frac{C_f \rho_a(r)}{\rho^0 d}\sqrt{GMr},$$

that gives the possibility to evaluate the critical number density n_{crit} (taking into account the relationships: $N = 4\pi r^2 \Delta h \rho$; $v_\tau = \sqrt{GM/r}$:

$$\rho > n_{crit} = \frac{3}{2\pi d^3} C_f \frac{\rho_a(r)}{\rho^0}\frac{r}{H(r)}\frac{1}{\Psi}. \tag{10}$$

The inequality (10) shows that the increase of production per one collision Ψ and particles diameter d is decreasing the critical number density; the increase of the atmospheric density and drag coefficient increases the critical number density. The critical number density decreases with altitude due to the exponential decrease of density suppressing the other altitude dependent multipliers in Eq. (10).

The estimate (10) shows that depending on the initial conditions the cascade process of debris growth due to its self-production could start independently at different altitudes. Since formula (10) is an approximate one based on a number of assumptions, it could provide mostly qualitative estimates. To obtain quantitative forecasts one needs to integrate the differential Eq. (6) for a number of phases ($j = 1, \ldots, N_p$) accounting for the mass exchange between the phases.

Another qualitative estimate of the long-term scenario of orbital debris evolution based on the inequality (10) is the following. The critical number density increases with the decrease of particles diameters ($n_{crit} \geq A/d^3$). Curve 1 in Figure 1 illustrates the qualitative behavior of the critical number density n_{crit} as a function of particle size (the scale is not given). The mean diameter of particles decreases in a fragmentation, the particles number density will increase. The last cannot grow faster than

$$\rho \leq \frac{6\alpha}{\pi d^3}, \tag{11}$$

in the absence of new launches.

The volume concentration of debris α decreases due to self-cleaning effect caused by the atmospheric drag. The evolution equation for α looks as follows:

$$\frac{\partial \alpha}{\partial t} = -W\frac{\partial \alpha}{\partial r} + \frac{\alpha W}{H(r)} - \frac{2\alpha W}{r}. \tag{12}$$

Assuming α is distributed uniformly within an altitude layer one obtains:

$$\frac{\partial \alpha}{\partial t} \sim \alpha W\left(\frac{1}{H(r)} - \frac{2}{r}\right) < 0 \tag{13}$$

$\partial\alpha/\partial t$ is always negative, because $W < 0$ and $H(r)/r \ll 1$.

After a characteristic time t_* after termination of space contamination, the decrease of α is given by

$$\alpha \leq \alpha_0 \exp\left(\frac{W}{H}t_*\right) = \alpha_0 \exp\left(-\frac{3}{2}C_f\frac{\rho_a(r)}{\rho^0}\frac{r}{H(r)}\frac{v_\tau t_*}{d}\right) = \alpha_0 \exp\left(-\frac{D_*}{d}\right), \tag{14}$$

where α_0 is the initial debris volume content following termination of external debris production.

The formulas (11) and (14) allow to obtain the upper estimate for the fragments number density accounting for self-cleaning:

$$\rho \leq \frac{6\alpha_0}{\pi d^3}\exp\left(-\frac{D_*}{d}\right). \tag{15}$$

Curve 2 in Figure 1 illustrates the upper limit for the fragments number density (unscaled qualitative illustration). The estimate (15) essentially depends on the value of debris volume

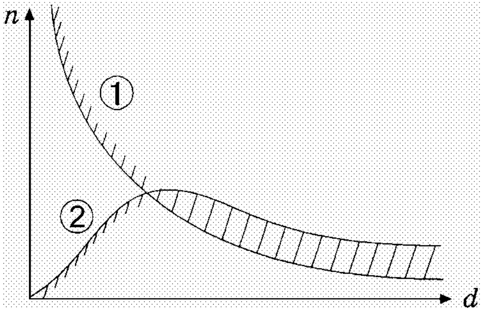

Figure 1. The upper and lower limits for the critical number density of orbital debris.

content α_0 at termination of external contamination. As it is seen from Figure 1, the decrease of the mean diameter of the fragments being the result of collisions finally makes the actual number density lower than the critical one for the smaller diameters (intersection of curves 1 and 2 in Figure 1). That causes the termination of the cascade process: self-cleaning surpasses self-production. Thus the analysis shows that in the long run after termination of space activities, self-cleaning will dominate in low orbits. The characteristic times necessary for the self-cleaning process to exceed the self-production could be estimated based on the solution of the unsteady problem using the full set of equations.

3. Evaluation of Collision Probability

The average number of collisions of spherical-shaped spacecraft (SC) in a circular orbit with small-sized space debris particles is determined as follows (Nazarenko, 1993, 1996, 1997):

$$\frac{dN}{dt} = S \cdot \rho(t) \cdot \int_{A=0}^{2\pi} p(t, A) \cdot V_{\text{rel}}(t, A) \cdot \mathrm{d}A = S \cdot \rho(t) \cdot \bar{V}_{\text{rel}}(t), \tag{16}$$

where S is the satellite cross-section, ρ is the spatial density of particles, $p(t, A)$ is their azimuthal distribution at time moment t and V_{rel} is the relative velocity of a particle with respect to the given satellite. The integral has a meaning of the mean relative velocity of a space object (SO) at this point.

The averaging of SO flux through a unit cross-section of a space vehicle is performed for one revolution (for the time interval equal to the SC period T). This mean value is calculated by the formula

$$\bar{Q} = \frac{1}{T} \cdot \int_{t=0}^{T} \rho(t) \cdot \int_{A=0}^{2\pi} p(t, A) V_{\text{rel}}(t, A) \cdot \mathrm{d}A \cdot \mathrm{d}t. \tag{17}$$

The study and estimation of probabilities of mutual collisions of objects belonging to different groups – large-size (catalogued), medium-size (from 1 up to 20 cm) and small-size (e.g. from 0.1 up to 1 cm) etc. – is of considerable interest. We shall assume that the space debris can have various sizes including those, which cannot be neglected. A possible size of particles will be characterized by the probability density of distribution $p(d)$ for their mean diameter d. Taking account of the distribution density $p(d)$, we can introduce into Eq. (16) modifications, which take into consideration the variability of particles' sizes. It is convenient to express the spatial density of particles, with a size larger than the arbitrary quantity d, as a product of some dimensionless factor $k(d)$ by the spatial density of particles with a size larger than some specified value d_0:

$$\rho(d, t) = k(d) \cdot \rho(d_0, t), \tag{18}$$

where the coefficient $k(d)$ is supposed to be independent of time. We designate the derivative of coefficient $k(d)$ as $f(d) = \mathrm{d}k(d)/\mathrm{d}d$. Then the averaged number of collisions of a SC, having size D, with particles whose size lies in the range of (d_1, d_2), can be expressed as

$$K(D, d_1, d_2) = F_d \cdot \bar{Q}(d_0, t) \cdot (t - t_0), \tag{19}$$

where

$$F_d = \left[-\frac{\pi}{4} \int_{d_1}^{d_2} (D+d)^2 \cdot f(d) \cdot \mathrm{d}d \right].$$

Having estimated the value $K(D, d_1, d_2)$ – the averaged number of collisions of a single spacecraft of diameter D with particle sizes in the range of (d_1, d_2) – one could develop the average number of collisions of a group of objects, having size in the range of (D_1, D_2) and situated in some altitude region $(h, h + \Delta h)$, with all SOs having size in the range of (d_1, d_2). This estimate is designated as $K(h, h + \Delta h)_{Dd}$ below. It is necessary to sum up the estimates $K(D, d_1, d_2)$ for all SOs of the given size lying in the given altitude range. As a result, we obtain the following estimate:

$$K(h, h + \Delta h)_{Dd} = F_{Dd} \cdot n(h, h + \Delta h)_{\mathrm{cat}} \cdot \bar{Q}(d_0, h, t_0) \cdot (t - t_0), \tag{20}$$

where F_{Dd} is calculated by the formula

$$F_{Dd} = \left[\frac{\pi}{4} \int_{D_1}^{D_2} \int_{d_1}^{d_2} (D+d)^2 \cdot \mathrm{d}k(D) \cdot \mathrm{d}k(d) \right] \Big/ 2. \tag{21}$$

This value has a meaning of the mean cross-sectional area of collisions of objects in the size range (D_1, D_2) with particles from the size range of (d_1, d_2), where $n(h, h + \Delta h)_{\mathrm{cat}}$ is the number of the catalogued objects within the altitude range $(h, h + \Delta h)$.

With this discretization (distinguishing finite number of phases) for the numerical modeling of continuous functions will bring to some errors in determining the characteristic values of the governing parameters. For evaluation of these errors one could use the following formula:

$$\chi_{\mathrm{error}} = \frac{\sum_{j=1}^{N_{\mathrm{p}}} p(d_j) \Delta \chi_j}{\chi_{\max} - \chi_{\min}} 100\%,$$

where $p(d)$ is the probability density function for particles *versus* size distribution $\int_0^{d_{\max}} p(d)\,\mathrm{d}d = 1$. In case of subdivision of parameters in groups with an equal step $\Delta\chi_j = \Delta\chi_k = \Delta\chi$ the formula for the error evaluation could be essentially simplified:

$$\chi_{\mathrm{error}} = \frac{\Delta\chi}{\chi_{\max} - \chi_{\min}} 100\% = \frac{100\%}{N_{\mathrm{p}}}.$$

The last equation shows that for an error not exceeding 10% one needs to introduce at least 10 phases. Taking into account non-uniformity of the $p(d)$ function, which allowed us to introduce non-equal steps, the same accuracy could be obtained with only eight phases. Thus the number of phases was chosen so that the error would not exceed 10%, which is consistent with the accuracy of our knowledge of the space environment.

In conclusion to this section we shall consider the results of calculations of the matrix F_{Dd} for the range of sizes presented in Tables I and II (here d_j is the left boundary of the jth size range).

TABLE I
The estimated values of $k(d)$ coefficient for phases $j = 1, \ldots, 10$

j	1	2	3	4	5	6	7	8	9	10
d_j, m	0.001	0.002	0.005	0.010	0.022	0.046	0.100	0.200	2.5	10
$k(d_j)$	8130.7	1302.6	208.7	33.4	11.9	4.3	1.5	1.0	0.2432	0.000075

TABLE II
Components of matrix F_{Dd} (m^2)

$j \backslash j$	1	2	3	4	5	6	7	8	9
1	137.6	54.8	27.7	14.6	21.8	34.0	29.3	1546	16181
2		16.3	6.42	2.82	3.83	5.57	4.80	248.4	991.3
3			1.90	0.63	0.73	1.00	0.80	40.0	159.1
4				0.15	0.13	0.15	0.11	4.98	19.7
5					0.09	0.08	0.05	1.83	7.08
6						0.05	0.03	0.70	2.58
7							0.01	0.15	0.52
8								0.56	1.04
9									0.86

Formula (20) and the F_{Dd} matrix (Table II) for SOs of different sizes lead to the following conclusion. The number of collisions of small-sized particles (smaller than 1 cm) between each other, as well as with larger objects, is much higher, than the number of mutual collisions of catalogued objects (size larger than 10–20 cm). This result testifies the necessity of taking into account mutual collisions of space debris of different sizes.

4. Analysing the Variety of Collision Scenarios

The space debris model (SDPA), applied in this study, allows to develop the following parameters, which influence the consequences of collisions:

- A variety of sizes of colliding particles, namely the sub-division of particle sizes under consideration into the groups presented above in Table I. The objects larger than 20 cm are combined below into one group ($j = 8$).
- A variety of masses of particles of the given size. The information on masses is contained in a two-dimensional distribution of a number of SOs *versus* size and ballistic factors (k_b). The ballistic factor represents here the ratio of cross-sectional area ($S = \pi d^2/4$) to the mass. Table III presents, as an example, the current distribution $p(k_b, d_j)$ at the altitude of 950 km. For each of possible values of SO sizes and ballistic factors the values of mass were calculated. They are presented below in Table IV. A typical feature of these data is the large range for variations of the possible values of parameters. The maximum of any value differs from its minimum as much as 10^7 times.
- A variety of possible impact velocities V_{rel}. The statistical distribution of possible values of these velocities $p(V_{rel})$ is constructed in the SDPA model on the basis of formula (16).

TABLE III
The current distribution of particles *versus* size and ballistic factor $p(k_b, d_j)$ at the altitude 950 km

j	Values of ballistic factors (m²/kg)					
	0.005	0.015	0.05	0.15	0.50	1.50
1	0.000	0.000	0.000	0.526	0.394	0.080
2	0.000	0.000	0.108	0.432	0.385	0.075
3	0.000	0.000	0.344	0.413	0.205	0.038
4	0.000	0.000	0.036	0.166	0.452	0.346
5	0.000	0.012	0.088	0.221	0.343	0.336
6	0.003	0.018	0.127	0.263	0.361	0.228
7	0.010	0.063	0.166	0.306	0.358	0.097
8	0.512	0.458	0.015	0.010	0.005	0.000

TABLE IV
Mass of objects (in grams) as a function of size and ballistic factor

j	Values of ballistic factors (m²/kg)					
	0.005	0.015	0.05	0.15	0.50	1.50
1	—	—	—	0.012	0.0035	0.0012
2	—	—	0.16	0.054	0.016	0.0054
3	—	—	0.73	0.24	0.073	0.024
4	—	—	3.5	1.2	0.35	0.12
5	—	54	16	5.4	1.6	0.54
6	520	240	73	24	7.3	2.4
7	2500	1200	350	120	35	12
8	4 18 000	1 91 000	57 000	19 100	730	—

Examples of such distributions for satellites with inclinations of 55°, 75° and 95° are presented in Figure 2.

- A variety of altitudes, where the collision can take place. In this paper the altitude range from 400 to 2000 km was considered, where about 80% of all large-size satellites were situated. The altitude distribution of a number of collisions is taken into account on the basis of formula (20).

5. Fragmentation Model for Hypervelocity Collisions of Space Debris Particles

The developed model allows the evaluation of collision probabilities, relative velocities, masses and sizes of colliding objects within all the altitude ranges. The results of collisions could be evaluated using the fragmentation model (Kiselev, 2001; Smirnov et al., 1997). The basic relationships for this model modified to meet the requirements of LEO debris self-production modeling are presented briefly below.

The high-velocity collision of particles of mass M_1 and M_2 is considered, the velocities of particles being equal V at the time of collision. The angle between velocity vectors is

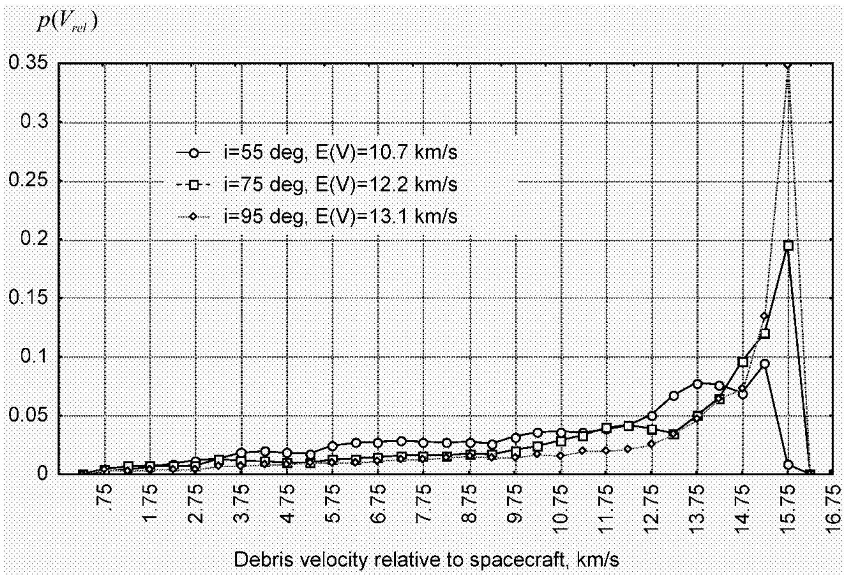

Figure 2. Probability density function of collision velocity $p(V_{\rm rel})$ for different orbit inclinations.

equal to 2β (in the inertial space). We use the designations: $M_\Sigma = M_1 + M_2, k_1 = M_1/M_\Sigma$, $k_2 = M_2/M_\Sigma$. Then the mean velocity of fragments after collision will be

$$V_M = V \cdot \sqrt{1 - 4 \cdot k_1 \cdot k_2 \cdot \sin^2 \beta}. \tag{22}$$

The amount of energy generated in collisions is characterized by the density of internal energy u, which is uniformly distributed within the particles. This specific internal energy can be determined as follows:

$$u = \frac{U}{M_\Sigma} = \frac{1}{2} \cdot k_1 \cdot k_2 \cdot (2 \cdot V \cdot \sin\beta)^2 = \frac{1}{2} \cdot k_1 \cdot k_2 \cdot (V_{\rm rel})^2. \tag{23}$$

In deriving this formula we used the assumption, that after collision all fragments (of total mass M_Σ) are moving with the same velocity (22). Indeed, with a great difference in size (and mass) between a target and a particle ($M_1 \ll M_2$) the energy of collision is absorbed only by a small fraction of target's mass (by individual components of the SC structure in our case). In this case in applying formula (9) only some portion of masses of colliding bodies M_2 should be used as a reference mass, rather than their total mass (Kessler, 1978), i.e.:

$$\mathrm{M}_2 = \begin{Bmatrix} \Gamma M_1, & \text{if } \Gamma M_1 \le M_2 \\ M_2, & \text{if } \Gamma M_1 > M_2 \end{Bmatrix}, \quad M_1 \ll M_2.$$

The density of internal energy is supposed to be sub-divided into elastic (e) and inelastic (dissipation) (ζ) components, that is, $u_\alpha = e_\alpha + \zeta_\alpha$ ($\alpha = 1, 2$). The entropic criterion of limiting specific dissipation is used here as a macrodestruction criterion, that is, $\zeta \le \zeta^*$, where ζ^* is the limiting specific dissipation, which depends on particle's material and is assumed to be known. (The term macrodestruction here means that an object is being split into separate fragments. The microdestruction, which means formation of defects and small internal cracks in the deformed body, could take place under much lower values of ζ.) It is also assumed, that a part of elastic energy, accumulated in particles after collision, is spent for destruction of particles, i.e., for formation of new free surfaces in solid matter. Then the energy spent for destruction can be calculated by formulas $e_\alpha^f = k \cdot e_\alpha$, where k is some factor, which is considered to be known. If the internal energy of a particle is found to be $u_\alpha < \zeta_\alpha$, then the destruction of particle α does not occur.

For the description of the fragments distribution in mass, the modification of Weibull distribution is used:

$$N(<m) = N_0 \cdot \left[1 - \exp\left(-\left(\frac{m - m_{\min}}{m_*}\right)^\Lambda\right)\right], \quad m_{\min} \le m \le m_{\max}, \tag{24}$$

where m_* is the characteristic mass of fragments distribution (to be determined in course of solution); Λ is the parameter, whose value depends on particle's material. It characterizes the degree of 'compactness' of destruction fragments' distribution in masses. For the discrete spectrum of particles' masses $m_1^\alpha, m_2^\alpha, \ldots, m_{K\alpha}^\alpha$ the number of fragments of ensemble m_j^α is

$$\begin{aligned} N_j^\alpha &= N_0^\alpha \cdot (b_j^\alpha - b_{j+1}^\alpha), \\ b_j^\alpha &= \exp\left(-\left(\frac{\sqrt{m_{j-1}^\alpha \cdot m_j^\alpha} - m_{\min}^\alpha}{m_*^\alpha}\right)^{\Lambda\alpha}\right). \end{aligned} \tag{25}$$

The system of K_α equations (25) is supplemented by the two following equations:

$$\sum_{j=1}^{K\alpha} m_j^\alpha \cdot N_j^\alpha = M_\alpha, \tag{26}$$

$$\sum_{j=1}^{K\alpha} \gamma_\alpha \cdot \frac{s_j^\alpha}{2} \cdot N_j^\alpha = M_\alpha \cdot e_\alpha^f, \tag{27}$$

where γ_α is specific energy required for producing a unit of free surface in destruction, s_j^α is the area of free surface, which was formed due to breakup, for fragment m_j^α ($s_j^\alpha \le S_j^\alpha$, where S_j^α is the total area of the external surface for the fragment m_j^α, which could partly contain the zones of external surface of initial fragment M_α). Equation (26) expresses the condition that the total mass of particle's fragments is equal to the initial mass of the particle, and Eq. (27) expresses the equality of elastic energy, accumulated in a particle, to the energy spent for producing a destruction surface. To express the relation between the destruction surface and fragment's mass, we introduce the dimensionless coefficient of fragment's shape characterizing its compactness: $(k_F)_j^\alpha = m_j^\alpha/(\rho_\alpha^0 \cdot (S_j^\alpha)^{3/2})$. Here ρ_α^0 is the specific mass of a particle and S_j^α is the total area of its surface. Obviously, the destruction surface s_j^α of a particle is a part of its total surface, that is $s_j^\alpha/S_j^\alpha = \theta_j^\alpha$, $0 < \theta_j^\alpha < 1$. Similarly, the dimensionless coefficient k_F cannot be greater than its value for a spherical element, as a most compact one, having the lowest surface for a fixed volume, that is $k_{F,j} = \xi_j \cdot (1/6\sqrt{\pi})$, $0 < \xi_j < 1$. With regard to this consideration, the relation between the destruction surface of a particle and its mass is as follows:

$$(s_j^\alpha)^{3/2} = \frac{m_j^\alpha}{\rho^\alpha} \cdot 6\sqrt{\pi} \cdot \frac{\theta_j^{1.5}}{\xi_j}. \tag{28}$$

The substitution of (28) into (27) results in a system of $K_\alpha + 2$ equations for each of colliding particles. The number of unknown values is also equal to $K_\alpha + 2$, namely N_j^α, $j = 1, 2, \ldots, K_\alpha$, N_0^α and m_*^α. The system of equations is non-linear with respect to the latter unknown parameter. Nevertheless, it can easily be solved by iterations. Thus the developed breakup model is a theoretical one, which uses some material properties and process characteristics derived from experiments. By now the described breakup criterion based on the energetic approach is the most advanced one, adopted by those working in mechanics of destruction. The detailed explanation for the model can be found in (Smirnov, 2002).

The initial data for the problem is listed below. It could be sub-divided into three groups:

- The first group includes values $V_{\rm rel}$, M_α and ρ_α^0, which characterize conditions of collision.
- The second group includes parameters ζ_α^*, γ_α and Λ_α, which characterize physical properties of particles' material.
- The third group includes parameters Γ, k, θ_α, ξ_j, $m_{\min}$, $m_{\max}$, which are specified on the basis of *a priori* data. For example, we accept below: $\Gamma = 115, k = 0.5, \theta_\alpha = 1, \xi_j = 1$ depending on the size (mass) of destruction fragments. Values $m_{\min}$, $m_{\max}$ are determined by approximate formulas in the process of iterations depending on the value of unknown m_*^α.

The solution of the system of Eqs. (25)–(27) with respect to unknown values listed above does not provide the final data on the consequences of collisions. In particular, the problem of the evolution of orbital parameters of fragments and their contribution into the population of space debris larger than 0.1 cm considered in the SDPA model (see Table I) remains unsolved. On solving the above problems we took into account:

(a) the mean value of fragment velocity at a collision (22);
(b) the velocity increments $v_j^\alpha = \sqrt{2 \cdot (1-k) \cdot e_\alpha}$, acquired by particles after collision;
(c) the relation between the area of particle's surface and its size: $d_j \approx \sqrt{s_j^\alpha/\pi}$.

As a result, the following parameters are calculated as the output data on application of the fragmentation model considered above:

- the number of generated fragments;
- the number and mass of fragments remained in orbit;
- the number and mass of fragments sizing larger than 0.1 cm;
- the distribution *versus* the perigee altitude for destruction fragments sizing larger than 0.1 cm.

As an example, we consider the results of modeling the collision of two spherical particles: a particle of steel (2 g mass) ($\alpha = 1$) and a particle of aluminum ($\alpha = 2$) having a mass of 20 g. The altitude of a circular orbit of objects before collision was 950 km. The values of $\sin\beta$, used in formulas (22) and (23), were taken to be equally probable over the interval of 0–1.0. Table V presents the data on the average number of objects of different size – both remained in orbit and deorbited due to the influence of atmospheric drag. In total,

TABLE V

Number of particles formed in collision of two spherical objects: steel (2 g mass) and aluminum (20 g mass) in a circular orbit 950 km altitude

No of an object	Range of fragments' size (the lower boundary only, cm)								
	0.0025	0.005	0.010	0.025	0.050	0.100	0.25	0.50	1.0
Remained on orbit									
$\alpha = 1$	5	486	28 401	43 053	1256	121	12	1	0
$\alpha = 2$	4427	39 976	2 17 271	5 47 138	7387	3716	452	16	1
Deorbited									
$\alpha = 1$	5	444	25 884	38 870	1069	96	9	1	0
$\alpha = 2$	4033	36 416	1 97 674	4 95 929	50 402	3086	346	10	1

TABLE VI

Number of particles of different sizes *versus* the perigee altitude formed in collision of two spherical objects: steel ball (2 g mass) and aluminum (20 g mass) in a circular orbit 950 km altitude

Altitude (km)	Range of fragments' size (the lower boundary only, cm)								
	0.0025	0.005	0.010	0.025	0.050	0.100	0.25	0.5	1.0
450	36	328	2010	5036	652	60	10	1	0
550	36	328	2010	5036	652	60	10	1	0
650	36	328	2010	5036	652	60	10	1	0
750	36	328	2010	5036	652	60	10	1	0
850	36	328	2010	5036	652	60	10	1	0
950	4253	38 825	2 35 620	5 65 013	55 383	3539	414	14	1
Sum	4433	40 465	2 45 670	5 90 193	58 643	3839	464	19	1

1 797 954 fragments of different size were formed and 943 717 (52%) of them continued orbital motion, the remaining 48% deorbited. It is seen, that the maximum of the size distribution of a number of fragments lies in the range of 0.025–0.05 cm. Table VI presents also the distribution of a number of objects of different size over the perigee altitude.

It is seen from Table VI that the overwhelming majority of fragments have a perigee altitude in the range of 900–1000 km. It is just the altitude, at which the space objects were orbiting before their collision. The aforementioned peculiarity is a consequence of the following fact: in the collision under consideration about a half of all fragments acquired positive increment of orbital velocity. The perigee altitude of these objects remained the same. Their apogee altitude increased.

The number of fragments, whose perigee altitude decreased, is rather small. It represents a part of the second half of all fragments with a negative velocity increment. The major part of them deorbited. The remaining fragments of this type have a uniform distribution of perigee altitudes.

6. Taking into Account the Consequences of Collisions in Space Debris Environment Modeling

In this section SO are considered with a perigee altitudes below 2000 km. It is assumed that among all variable SO parameters only the perigee altitude essentially influences the evolution of the altitude distribution. The other orbital elements will be designated by χ. We subdivide the whole set of objects with different elements χ into some finite number of sub-sets (groups) with elements χ_i, $j = 1, 2, \ldots, i_{\max}$. Then we state the problem of studying the laws of time variation for fragments density of distribution $N_j(t, h)$ by solving Eq. (6). Subscript j will be omitted hereafter in analyzing the evolution of distribution for some particular SO group.

In calculating the evolution of the altitude distribution of SO number the following factors were taken into consideration:

- the atmospheric drag at altitudes up to 2000 km;
- the sub-division of all SOs into the groups, which differ in size d, eccentricity ε and ballistic factor k_b (χ_i parameters);
- the initial altitude distribution of SOs of various types;
- the expected rate of formation of new SOs of various types as a result of launches and explosions: $\dot{N}_{j\Sigma}(t, h)$ is the increment of different SO number at various altitudes per time unit.

The technique for semi-analytical solution of Eq. (6) was developed for the SDPA model. We designate by 'SDPA-STEP' the procedures of making one step in time in solving Eq. (6) for particles of the given size. Such a structure of the algorithm was based on the assumption that the rate of formation of new SO of various types $\dot{N}_{j\Sigma}(t, h)$ does not depend on the current level of population. This assumption was true for the model taking into account debris formation due to new launches only. The details are given in Nazarenko (1993, 1996, 1997) and Smirnov (2002).

Taking collisions into account requires essential updating of the algorithm. The basic details of this updating are considered below. They take into account the important feature of the SDPA model – the statistical approach to the description of state and sources of the near-Earth space environment.

The diversity of collision conditions, which essentially influence the consequences of collisions, plays the key role here. Numerical modeling needs definite discretization, which, nevertheless, should preserve the major characteristic features of the regarded processes. For example, the altitude range of 1600 km should be discretized at least with an interval 100 km to provide an adequate accuracy of description, which requires 16 different altitude layers. The adopted sub-division of all fragments into eight phases leads to 36 collision scenarios at each altitude layer, etc. Table VII presents the basic factors influencing the

TABLE VII

Minimal number of scenarios to be regarded in discretized numerical modeling of space debris evolution and self-production in LEO

No	Governing parameters	Number of versions
1	Size range of colliding SOs, d_j, $j = 1, \ldots, 8$	$1 + 2 + \cdots + 8 = 36$
2	Altitude of collision, h	$1600/100 = 16$
3	Ballistic factors (masses) of SO, k_b	$6 * 6 = 36$
4	Collision velocity (angle between the vectors), V_{rel}	45
	The total number of versions is equal to $36 \cdot 16 \cdot 36 \cdot 45 = 933120$	

debris evolution and self-production processes and the minimum number of scenarios to be regarded in discretized numerical modeling.

If we sort out all these versions in the process of integration of the equations (at each time step), the algorithm could not be implemented on a modern personal computer.

The use of statistical distributions of SOs *versus* altitude, ballistic factors and velocity allows us to determine the averaged consequences of one collision of SOs of different sizes (36 versions) beyond the solution of a forecasting problem. Though the calculations of these consequences are rather time consuming, they are executed only once – at a preparatory step to the forecasting procedure. Further on, during the integration of Eq. (6), the matrix of probabilities of collision of SOs of different sizes is calculated at each time step. In this symmetrical matrix P_{Dd} (8×8) only 36 values are meaningful. Their multiplication by *a priori* calculated characteristics of collision consequences allows us to determine the component of the sum $\dot{N}_{j\Sigma}(t, h)$, which relates to collision consequences, as well as some other parameters.

And, finally, one more updating of the SDPA model algorithm is relevant to the change of the orders of cycles in time and in size.

Figure 3 presents the enlarged flowchart of the environment forecasting algorithm, which takes into account collisions.

7. Collisions Contribution to the Current State of Space Debris Environment

The model forecast was performed for the time interval 1960 till 2000. The forecasts were made with and without mutual collisions of SOs sizing larger than 0.1 cm. In addition, the version of ‘partial collisions’ was considered, in which the collisions of all SOs except catalogued ones were taken into account. For these versions the data on a number of SOs of different sizes in 2000 are presented in the upper two lines of the Table VIII. Naturally, in case the collisions were taken into account, the number of small-sized space debris particles happened to be greater than that obtained in forecasts disregarding collisions (the third line of Table VIII). Considerable changes were observed only for particles sizing larger than 0.1–0.50 cm. Taking account of collisions the estimated number of particles was 18–22% greater, than without collisions. In the case of ‘partial collisions’ the estimates have had intermediate values. The data of Table VIII show, that the consequences of collisions of SOs of different sizes on a preceding time interval resulted in 11–12% increase of the number of particles in the size range 0.1–0.5 cm. The influence of this source on the population of large-size space debris is insignificant. The estimation of the contribution for particles with a size smaller than 0.1 cm requires additional analysis.

Figure 4 compares the altitude distributions of a number of SOs in the 100-km altitude bands in 2000, calculated using the model with and without collisions of SOs of different sizes, as well as in the intermediate case (without collisions of objects sizing larger than 20 cm). These data indicate that the maximum contribution of collision consequences is in the altitude range of 800–1000 km, which is explained by maximum number density of fragments in the present altitude layer. At higher altitude a sharp decrease of density decreases the collision probability, which, naturally, decreases the growth of collision fragments.

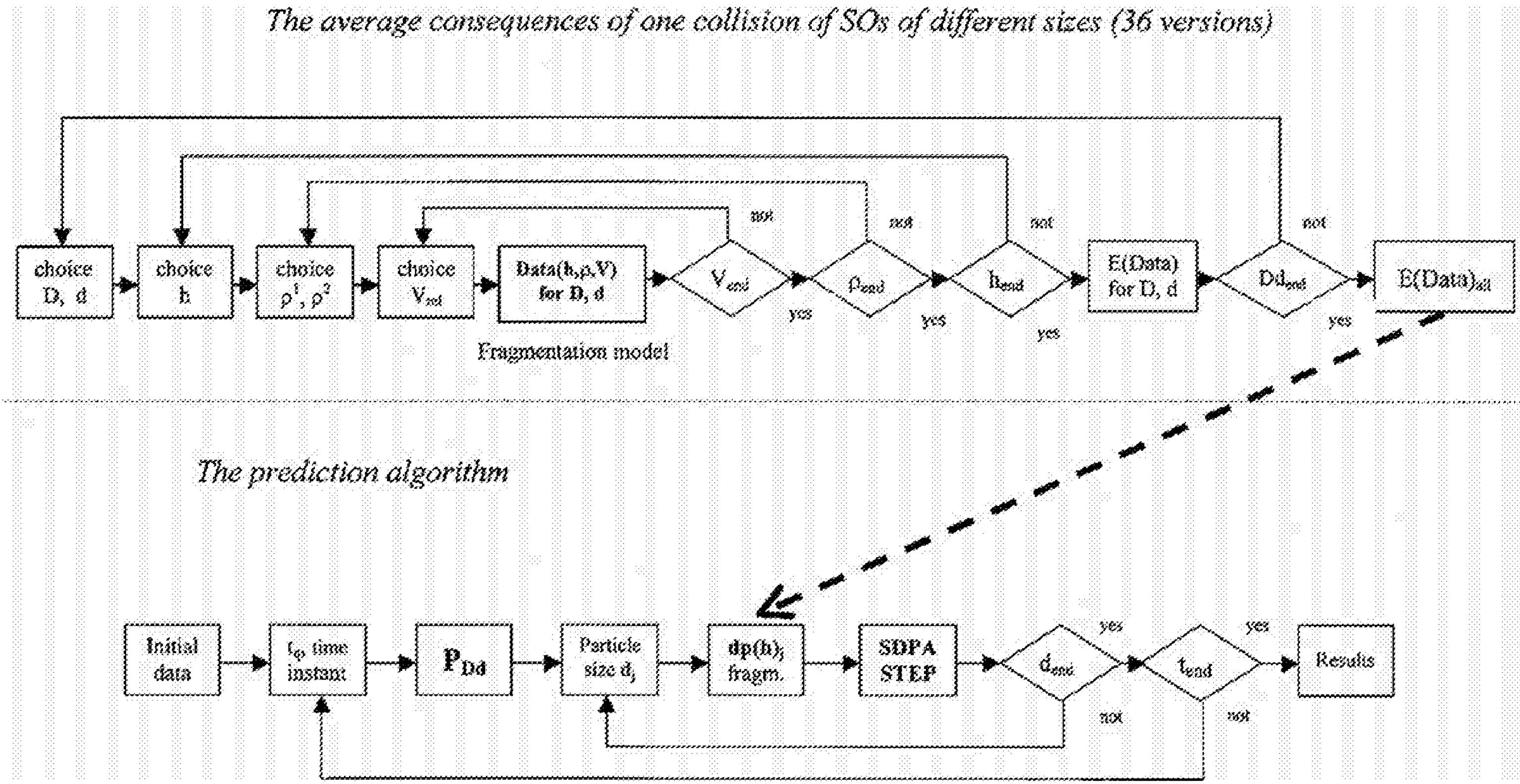

Figure 3. Flowchart for the prediction algorithm accounting for collisions of different types.

TABLE VIII

Forecasts for the time interval (1960–2000); number of particles of different sizes

Version	Size of particles (cm)							
	0.1–0.25	0.25–0.5	0.5–1.0	1.0–2.5	2.5–5.0	5.0–10	10–20	>20
All collisions	77.7E + 6	7.57E + 6	1.58E + 6	203 000	81 850	32 500	16 780	7699
Partial collisions	66.2E + 6	6.75E + 6	1.56E + 6	201 000	81 730	32 480	16 780	7700
No collisions	65.7E + 6	6.21E + 6	1.55E + 6	200 000	81 710	32 480	16 780	7700

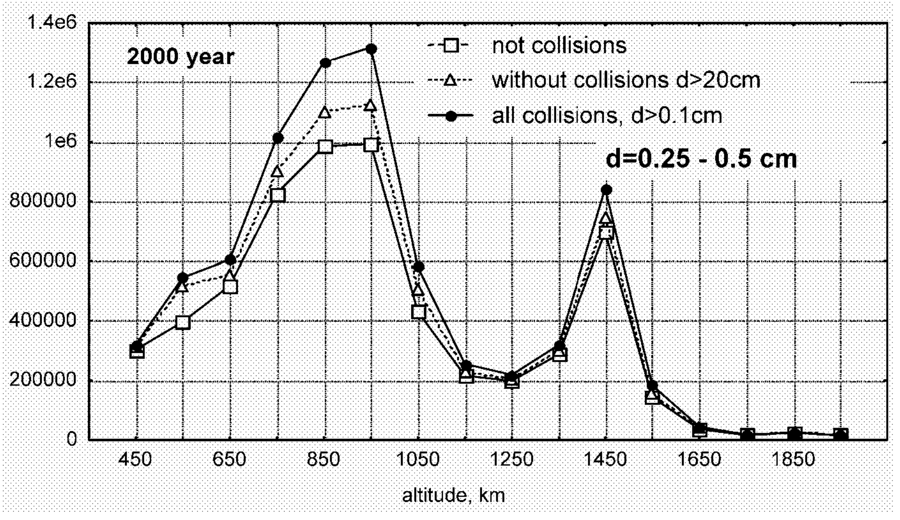

Figure 4. Comparison of the altitude distribution of the number of space objects for different prediction strategies in a 40-years forecast: AD 1960–2000. The space objects are given per 100 km altitude band.

The further growth of fragment number density with the second maximum at the altitude 1500 km increases the collision probability in that altitude layer and, as a consequence, causes an essential increase of collisions contribution into an overall number of the current debris population.

The maximal contribution of collisions with allowance for all mutual collisions in the altitude range of 800–1000 km is now equal to 33% of the total level of the altitude layer contamination with particles of the regarded size. The growth is 16% as compared to the intermediate case. This shows that the contribution of mutual collisions of catalogued objects is slightly greater, than the contribution of all other collisions under consideration. Nevertheless, the contribution of collisions of smaller SOs between each other and with large-sized objects is rather significant – it is equal to 14%.

As the technogeneous contamination level on a preceding time interval grows, the probability of mutual space debris collisions also increases. Table IX presents the estimates of a total (accumulated) expected number of collisions for the preceding time interval (1960–2000).

TABLE IX
Matrix of accumulated probabilities (the average number) of collisions for SOs of different sizes on the preceding time interval (years 1960–2000)

Size (cm)	$j = 1$ 0.1–0.25	$j = 2$ 0.25–0.5	$j = 3$ 0.5–1.0	$j = 4$ 1.0–2.5	$j = 5$ 2.5–5.0	$j = 6$ 5.0–10	$j = 7$ 10–20	$j = 8$ >20
$j = 1$	41.0	18.3	12.5	4.90	7.9	11.4	20.60	5490
$j = 2$		1.5	1.6	0.50	0.73	1.00	1.78	463
$j = 3$			0.32	0.16	0.20	0.25	0.42	105
$j = 4$				0.01	0.026	0.026	0.041	9.24
$j = 5$					0.009	0.015	0.019	3.58
$j = 6$						0.004	0.009	1.18
$j = 7$							0.003	0.52
$j = 8$								0.96

TABLE X
Characteristics of fragments generated in collisions on the preceding time interval (years 1960–2000)

Version	Mass of fragments (kg)			Number of SOs of different sizes (cm)			
	M_{sum}	$M(d < 0.1)$	$M(d > 0.1)$	0.1–0.25	0.25–0.5	0.5–1.0	1.0–2.5
Partial collisions	45.5	23.1	13.5	1.3E + 6	1.3E + 6	22 000	1100
All collisions	432	203.9	41.0	29E + 6	3.4E + 6	81 000	7000
Ratio, %	10.5	11.4	32.9	4.5	38.2	27.2	15.7

These data indicate that the greatest number of collisions (5490) could have occurred between particles sizing 0.1–0.25 cm and the catalogued SOs. The accumulated number of collisions for the catalogued SOs between each other on the preceding time interval is relatively low – it is equal to 0.96.

Now we consider the data on variations of some additional characteristics on the forecasting interval. These data include:

- the total mass of generated fragments (M_{sum});
- the mass of fragments remained in orbit (at the generation time instant) and sizing smaller than 0.1 cm ($M(d < 0.1)$);
- the mass of fragments remained on orbit (at the generation time instant) and sizing greater than 0.1 cm ($M(d > 0.1)$.

Table X presents the data for the parameters listed above in 2000. In addition, the estimates of the total number of generated fragments for the size ranges 0.1–0.25, 0.25–0.5, 0.5–1.0 and 1.0–2.5 cm (including deorbited objects) are presented.

With allowance for all possible collisions of SOs with a size larger than 0.1 cm the total mass of fragments equals 432 kg, on the average. About half of these fragments (43% in mass) deorbited at the time of collision. The other set of fragments (47% in mass) relates to small-sized particles smaller than 0.1 cm. And only a small part of mass (9%) relates to particles larger than 0.1 cm. With partial accounting of collisions (without catalogued SO) the total mass of small-sized debris equals to 11% of the respective estimate with account of all collisions. For larger fragments (>0.1 cm) this portion is much greater – 33%. This effect could be explained as follows: the density of internal energy Eq. (23) in mutual collisions of large SOs is greater, than that in collisions of small-sized particles with large SOs. This results in SO splitting into smaller fragments.

The total number of collision fragments of the size 0.1–0.5 cm (Table X) account for 45–55% in relation to the current number of fragments of this size estimated disregarding collisions (Table VIII). Thus, the contribution of collisions to the current population of particles in the size range 0.1–0.5 cm is rather essential.

8. Conclusions

The developed mathematical model for space debris evolution is based on a continuum approach, which is an alternative to the classical celestial mechanics approach the existing debris evolution models are based on. The model is able to follow the evolution of multi-component debris clouds incorporating classes of fragments of different types.

The analysis of the marginal cases made it possible to derive a simplified criterion determining the critical number density characterizing the beginning of the cascade process of debris self-production in collisions. It was shown that collisions of different types of fragments contribute not only to debris self-production, but to a self-cleaning of the LEO, because fragments of small diameters are much stronger influenced by the atmospheric drag. Taking into account this effect the role of collisions in the cascade effect of space debris growth in LEO should be thoroughly reconsidered.

The numerical technique of space debris evolution modeling based on the continuum approach is developed, which takes into account the consequences of collisions of objects of different sizes. The evolution model includes the following major components: the fragmentation model for high-speed impacts, the model evaluating the average collision consequences, the calculation of the matrix of SO mutual collisions probabilities and the numerical algorithm for solving differential equations in partial derivatives.

Model forecasts of the orbital debris population evolution on the preceding time interval of 40 years allowed to compare the results with the current state of space contamination thus validating the developed model. Comparison of the results for fragments larger than 20 cm with the current distribution of traceable SO provides good agreement. Comparison of results for fragments in the size range 1–20 cm shows that the results based on present model fall on the upper boundary of the existing estimates. Results for fragments in the size range 0.1–1.0 cm agree with the existing estimates.

The mean contribution of SO collisions to the catalogued SD environment has been estimated. Objects larger than 0.1 cm at altitudes up to 2000 km have been considered. It is found that the maximal contribution of collisions is reached in the altitude range of 800–1000 km with account of all mutual collisions and is equal now to 33% of the general contamination level of this high-altitude layer by particles sizing 0.25–0.5 cm.

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